International Journal of Mechanical and Industrial Technology ISSN 2348-7593 (Online)
Vol. 4, Issue 1, pp: (31-37), Month: April 2016 - September 2016, Available at: www.researchpublish.com

Mathematical Analysis of the Structure and Performance of an Automated Bread Slicing Machine

Emmanuel Obinna Okeke

Department of Mechanical Engineering, Federal University of Technology, PMB 1526, Owerri, Nigeria

Abstract: **This work assesses the performance of an automated bread slicing machine in comparison to a manually operated type by subjecting them to a performance test. Calculation of forces acting on the cutting components are presented and materials selection for optimal performance explained. The resultant slicing capacity of the automated machine show that average loaves sliced per minute are comparatively higher and crumbs of bread produced are smaller. The results obtained are presented using graphs that compare mass of crumb formation, tool cutting force against cutting speed and cutting speed against time. The results show that the manually operated bread slicing machine produce greater crumbs of bread as the cutting speed increases. The tool cutting force remained constant with increase in cutting speed and decreased gently as time elapsed.**

Keywords: **Bread slicing, Calculation, Optimal, Materials selection, Crumbs, Cutting force, Cutting speed.**

1. INTRODUCTION

Bread is one of the most common types of food globally consumed in large quantities on daily basis owing mainly to the widespread availability of its raw materials and market for the finished product. It can be found in various forms and is mainly made up of a cereal substance known as wheat. In most cases, bread is consumed with supplements such as beverages, jam and margarine.

Bread slicing is an act that dates to as far back as the 20th century when Otto Frederick Rohwedder from Iowa, Davenport in the U.S pioneered the act of mechanized bread slicing (*The National Museum of American History*). Researchers that followed have modified the earlier work done by Otto to come up with more efficient automated bread slicing machine.

2. MAJOR MACHINE COMPONENTS AND MATERIALS

The machine components are discussed under the following:

1. Body frame
2. Blade holder
3. Bread guide
4. Bread support
5. Blade
6. Pulley system
7. Bread conveyor system
8. Specific dimensions and capacity of the cutter motor
9. Materials selection

BODY FRAME:

Length of frame, L = 520mm

Width of frame, W = 425mm

Height of frame, H = 1103mm

Area of frame, A = 520mm x 425mm = 221 x 10^3mm^2

The approximate total centre weight of body frame is as calculated below:

Approximate total centre weight of frame (weight at centre of gravity) = weight of frame + weight of bread support + weight of blade holders + weight of conveyor + weight of cutter motor.

Approximate total centre weight = 400N + 50N + 20N + 50N + 250N = 770N

Weight of speed selector motor = 100N

Weight of pulley = 10N

Total downward forces = approximate total centre weight of frame + weight of speed selector + weight of pulley.

1a. Determining the reaction forces:

To ensure equilibrium, sum of action forces = sum of reaction forces.

Sum of action forces = 770N + 10N + 100N = 880N

$R_A + R_B$ = 770N + 10N +100N = 880N…………………………………………………….. (1)

R_A and R_B represent the reaction forces at points A and B of the body frame respectively.

Taking moment about B, $\sum M_B = 0$…………………………………………………………. (2)

R_A x 520mm – (770N x 180mm) – (10N x 20mm) – (100N x 60mm) = 0………………….. (3

Hence $0.52R_A = 144.8$

Therefore R_A = 278.46N

From equation 1; $R_A + R_B$ = 880N, therefore $R_B = 1245-R_A$ = 880N – 278.46N = 601.54N

1b. Determining the bending moments:

Bending moment = R(x)…………………………………………………………..... (4)

Where R = reaction force and x = distance between force and perpendicular distance.

Bending moment at C = $R_A(x)$ = 278.46N x 260mm = 72399.6KN-mm; approximately 72kN-mm

Bending moment at D = $R_B(x)$ = 601.54N x 80 = 48123.2N-mm; approximately 48kN-mm

Bending moment at E, $R_B(x)$ = 601.54N x 60 = 36092.4kN-mm; approximately 36kN-mm

Therefore maximum bending moment is at C.

BLADE HOLDER:

The blade holder comprises of the blade frame within which the cutting blades make their translational movement. The blade frame is dimensioned such that it is wide enough to accommodate blade movement without any interference during the cutting operation. Hence it is 410mm long, 320mm high and rectangular in shape. It is made up of a hollow inner rectangle and an outer solid rectangle.

Length of blade frame, L = 410mm = 0.41m

Height of blade frame, H = 320mm = 0.32m

Area of blade frame, A = L x H = 0.41m x 0.32m = $0.1312m^2$

To ensure any possible irregularity is adequately accommodated, an allowance of 0.1m is added to each of the length and height. Therefore the maximum permissible area, A_{max} = (L + 0.05m) x (W + 0.05m). Therefore A_{max} = (0.41m + 0.0005m) x (0.32m + 0.0005m) = 0.4105m x 0.3205m = 1.32 x $10^{-1}m^2$. Mass of frame = 2kg; hence Weight, W of frame is approximately 2kg x $10ms^{-2}$ = 20N.

Hence δ_{max} = 20N/0.1315653m^2 = 152.02Nm^{-2}

Factor of Safety (F.S) = 1.5

Hence Allowable Strength, $\delta_a = \delta_{max}$/F.S = 150.02Nm^{-2}/1.5 = 101.35Nm^{-2};

(*R. Khurmi & J. Gupta, 2006*).

Compressive force, $F_c = \delta_a$ x A_{max}….. (6)

Therefore F_c = 101.35Nm^{-2} x 0.1315653m^2 = 13.33N.

BREAD GUIDE:

Length, L = 279mm = 0.28m; Width, W = 5mm = 0.006m; Height, H = 30mm = 0.03m; Maximum distance between guides = 482mm = 0.482m (adjustable); Number of guides = 4; Arrangement: two at the delivery section and two at the feed-in section.

BREAD SUPPORT:

Full length = 510mm; Full width = 80mm; Mass of bread support = 0.5kg and g = 9.8ms^{-2}; Weight = mg = 0.5kg x 9.8ms^{-2} = 4.9N. Bending occurs mostly at the centre and this is as a result of the centrally situated earth's gravitational pulling (the resultant force acts at the centre). Centralizing the resultant weight ensures an equilibrium is maintained during the bread slicing process and this ensures the bread loaves do not slide out of position.

BLADE:

Length of each blade, l = 250mm; Width, w = 3mm; Thickness, t = 1mm; Number of blades per blade holder, n = 10. Space between blades is approximately length of blade frame divided by number of blades per blade holder; Length of blade frame = 410mm, number of blades = 10, hence spacing = 410mm/10 = 41mm. Mass of driven pulley, m = 4kg; Force = mass x acceleration; acceleration of motor = 420ms^{-2}, therefore Force, F on pulley = 4kg x 420ms^{-2} = 1680N.

Mass, m of each blade = 0.0255kg; weight, w = 0.0255kg x 10ms^{-2} = 0.25N

Therefore weight of ten blades = 0.25N x 10 = 2.5N.

Force, F on the driven pulley = 1680N

Sum of reaction forces equals force on driven pulley; therefore $R_1 + R_2$ = 1680N…………… (7)

Taking moments O;

$R_1 = R_2$……………………………………………………………………………………...….. (8)

R_1 x 205 = R_2 x 205

Substituting R_2 in equation 7; $R_1 + R_1$ = 1680N

$2R_1$ = 1680N; therefore R_1 = 1596/2 = 840N.

From the diagram above for reaction forces, each of the two vertical reaction forces makes an angle of 30^0 with the diagonal reaction force, R.

Hence $R_1/R = \cos30^0$…………………………………………………………………..... (9)

Therefore R = $R_1/\cos30^0$ = 840/0.8660 = 969.94N.

To calculate the horizontal component of the reaction force;

$R_h/R_1 = \tan30^0$…………………………………………………………………………….... (10)

$R_h = R_1\tan30^0$ = 969.94 x 0.5774 = 560N.

5a. Shear force calculation for blades:

Shear force calculation between points A and O ($0 < x < 205$)

Shear force $Q = Rw_x$ where R = Reaction force and w_x = uniformly distributed load across length x.

$Q_0 = R = 969.94N$ (at x = 0)

$Q = 4 \times 205 = 820N$ (at x = 205)

$Q_A = 969.94N - 820N = 149.94N$.

$Q_B = 0 - w_x + R_2 = -w_x + R_2$

$Q_B = (-4 \times 412) + 969.94 = -1648 + 969.94 = -678.06$.

5b. Bending moment calculations for blades:

Between points O and A, bending moment BM: $BM_x = Rx - Wx.x/2$

$BM_x = R.x - Wx^2/2$; at x = 0, $BM_A = 0$. At x = 205mm; $BM_O = (820 \times 205) - (4 \times 205^2)/2 = 168100 - 84050$kN-mm; approximately 84kN-mm. Distance between points A and B, x = 410mm; $BM_B = (820 \times 410) - (4 \times 410^2)/2 = 336200 - 336200 = 0$.

Hence maximum bending moment, BM_{max} is at x = 205mm and is equivalent to 84KN-mm.

PULLEY SYSTEM:

Diameter of larger pulley (driven), D_L = 131mm = 0.131m

Radius of larger pulley, $R_L = D_L/2 = 0.131m/2 = 0.066m$

Diameter of smaller pulley (driver), D_S = 51mm = 0.051m

Radius of smaller pulley, $R_S = D_S/2 = 0.051/2 = 0.026m$

Speed of driver pulley, N_1 = 200rpm

Distance between the centres of the two pulleys, x = 500mm = 0.5m

Maximum tension in belt, T = 1000N; Coefficient of friction, $\mu = 0.25$; thickness of belt, t = 0.005m.

Length of belt, $L = \pi (R_S+R_L) + 2x + (R_S+R_L)^2/x$ (*Hall et al, 1971*).

$L = 3.142(0.026+0.066) + 2(0.5) + (0.026+0.066)^2/0.5 = 0.289 + 1 + 0.00423 = 1.293m$.

Angle of contact between belt and pulley, $Ø = (180^0-2\alpha)$ where $\alpha = \sin^{-1}(R_L-R_S)/x$ (*W. Chapman, 1975*).

Hence α = sin-1(0.066-0.026)/0.5 = $\sin^{-1}(0.08) = 4.59^0$, therefore $Ø = (180^0-2x4.59^0) = 170.82^0$

This value is equivalent to $170.82^0 \times \pi/180$ radians = 2.98 radians.

Velocity Ratio of pulley: $N_2/N_1 = D_L/D_S = 0.131m/0.051m = 2.57$.

Velocity Ratio considering thickness of belt: $N_2/N_1 = (D_L + t)/(D_S + t) = (0.131m + 0.005m)/(0.051m + 0.005m) = 0.136m/0.056m = 2.43$.

Velocity of belt, $V = \pi DN/60$ (*R. Jain, 2009*); hence Velocity, $V = \pi \times 0.131 \times 200/60 = 1.37\ ms^{-1}$

In the absence of slips, $V_1 = V_2 = 1.37ms^{-1}$.

Ratio of driving tensions is stated as $2.3\log (T_1/T_2) = \mu Ø$ (*R. Khurmi & J. Gupta, 2006*);

$\mu = 0.25$, $Ø = 2.98$rad/s hence $\log (T_1/T_2) = (0.25 \times 2.98)/2.3 = 0.324$.

This implies that $T_1/T_2 = \log^{-1}(0.324) = 2.11$.

Tension $T = T_1 = 1000N$ (assuming no centrifugal tension), therefore $1000/T_2 = 2.11$

This implies that $T_2 = 473.93N$.

Torque in driven pulley, $\tau_2 = (T_1-T_2) \times 0.026m = 13.68$ Nm.

Power transmitted by the belt, $P = (T_1-T_2) \times V_1$ (*Hall et al, 1971*);

P= 526.07N x 1.37ms-1 = 720.7 watts

6a. Pulley shaft:

Torque, $\tau_S = 60P/2\pi N_2$ (*J. Chapman, 1975*); $N_2/N_1 = 2.57$, $N_1 = 200$rpm, therefore $N_2 = 200$rpm x 2.57 = 514rpm.

Power, P = 720.7 watts, hence $\tau_S = (60 \times 720.7)/(2 \times 3.142 \times 514) = 13.39$ Nm.

Moment, $M = (T_1 + T_2 + 2T) = (1000 + 473.93 + 2000) = 3473.93$ Nm.

Equivalent twisting Moment, $M_e = \sqrt{(\tau_S^2 + M^2)} = \sqrt{(179.2921 + 12068189.6449)} = \sqrt{12068368.937} = 3473.96$ Nm.

BREAD CONVEYOR SYSTEM:

Length, l = 765mm, Breadth, b = 520mm, length of 4 rollers put together = 420mm, 38.2mm diameter.

Speed of sprockets = 200rpm, Diameter of pitch circle, D = 6cm = 0.06m, Number of teeth on sprocket, T = 14.

Pitch, p = D x sin (360/2T) (*J. Shigley, 1989*); p = 0.06 x sin (360/28) = 0.0134m, Number of sprockets, n = 2, centre distance between two sprockets, x = 20cm = 0.2m;

Average velocity of the chain, $V_{AVR} = \pi DN/60 = TPN/60 = 14 \times 0.0134 \times 200/60 = 0.625\text{ms}^{-1}$.

Number of chain links is given as $K = (T_1+T_2)/2 + 2x/p + [(T_2-T_1)/2\pi]^2.p/x$ (*P. Sharma, 1982*);

$K = (14+14)/2 + 2(0.2)/0.0134 + [(14-14)/2\pi]^2 \times 0.0134/0.2 = 14 + 29.85 + 0 = 43.85$, approximately 44.

Length of chain link, L = Kp = 44 x 0.0134m = 0.5896m.

Power transmitted by chain is given as $P = (W_B \times V)/(n \times K_S)$ (*J. Shigley, 1989*); where W_B = breaking load = 0.3N, Safety factor, n = 1.5; Service factor $K_S = K_1 \times K_2 \times K_3$ where $K_1 = 1$ for constant load, $K_2 = 1.5$ for periodic lubrication, $K_3 = 1.5$ for continuous service. Hence $K_S = 1 \times 1.5 \times 1.5 = 2.25$; therefore Power, P = (0.3 x 0.641) / (1.5 x 2.25) = 0.057W.

SPECIFICATIONS AND CAPACITY OF THE CUTTER MOTOR:

The cutter motor is a single phase AC motor rated 220-240v, 50HZ, 10A, 40/50µF *(ASTM F2 646-07*); Speed of motor = 420rpm, Power rating = 1.5kW, Shaft diameter = 2.5cm = 0.025m, Length of shaft = 38cm = 0.38m.

Angular velocity of motor, $\omega = 2\pi N/60 = (2 \times 3.142 \times 420)/60 = 43.988$ rad/sec, approximately 44 rad/sec.

Area of shaft, $A = \pi D^2/4 = 3.142 \times 0.025^2/4 = 4.909 \times 10^{-4}\text{m}^2$; Torque, T transmitted by shaft = $(P \times 60)/2\pi N = (1500 \times 60) / 2 \times 3.142 \times 420 = 34.1$ Nm.

MATERIALS SELECTION:

The selection of material is as tabulated below:

S/N	COMPONENTS	MATERIALS	DIMENSIONS(mm)	REMARKS
1.	Main Frame	Mild steel	520 x 765 x 1233	Easily affordable and weldable.
2.	Cutting blades	Stainless steel	250 x 3 x 1	Corrosion resistance; does not contaminate bread.
3.	Blade frame	Stainless steel	410 x 320 x 20	Strong enough to with reaction forces; machinable.
4.	Blade guide	Iron	765 x 5 x 40	Relatively cheap; easy to weld and position.
5.	Bread support	Mild steel	510 x 80	Corrosion resistant; provides desirable weight balancing.
6.	Pulley	Mild steel	131(diameter) 30(belt space)	Cheap and available. Portable.
7.	Conveyor	Iron and steel	765 x 420 x 38.2	Easily workable and available.
8.	Sprockets	Alloy steel	45(base circle); 7.5(teeth height)	Easily affordable and available.

(Source: ASTM F2 646-07 Standard Specifications for Bread Slicing Machines)

3. PERFORMANCE TEST AND RESULTS

A manually operated bread slicing machine and an automated bread slicing machine were set up for bread slicing operation. There are 24 pairs of cutting blades held in position by two blade holders. The cutting speed is selected based on the blade material hence mild steel is used in this case and it worked at an average of 30 meters per minute. Bread loaves were fed to the cutter based on three fed rates. Different bread textures were obtained as the feed rates were periodically varied. Cutting speed and feed rate together determined depth of cut and material removal rate which is are also described as the volume of work piece material removable per unit time.

Efficiency of the automated bread slicing machine was calculated using: Efficiency = (work output)/(work input) x 100. Work output is calculated as force x distance. Hence work output Fd = Fx_1 where x_1 = distance covered by the blade. But force F = mass x acceleration and velocity = displacement/time = x_1/t. Distance covered by blade x_1 = 0.4m. Hence velocity = 0.4m/1 (for every second) = 0.4 ms^{-1}. Acceleration = v/t = 0.4/1 = 0.4 ms^{-2} (for every second). Mass of blade = 0.04kg, therefore force = 0.04kg x 0.4ms^{-2} = 0.016N. This implies that work output = 0.016N x 0.4m = 6.4 x 10^{-3}Nm. Work input = force x distance covered by bread. Distance covered by bread = x_2 = 0.383m. Mass of bread used m_2 = 0.007Kg, optimum acceleration a_2 of the bread was 2.48ms^{-2}. The optimum acceleration component of the bread was resolved into $a_2$sinØ where Ø was the angle of feed and equivalent to 90^0. Hence force = $m_2a_2\sin90^0$ = 0.007Kg x 2.48ms^{-2} x $\sin90^0$ = 0.01736N. Therefore work input = 0.01736N x 0.383m = 6.649 x 10^{-3}Nm. This implies that efficiency of the automated system = 6.4 x 10^{-3}Nm/6.649 x 10^{-3}Nm = 0.9626 = 96.26%. Below is a graphical analysis of mass of crumb formation against cutting speed.

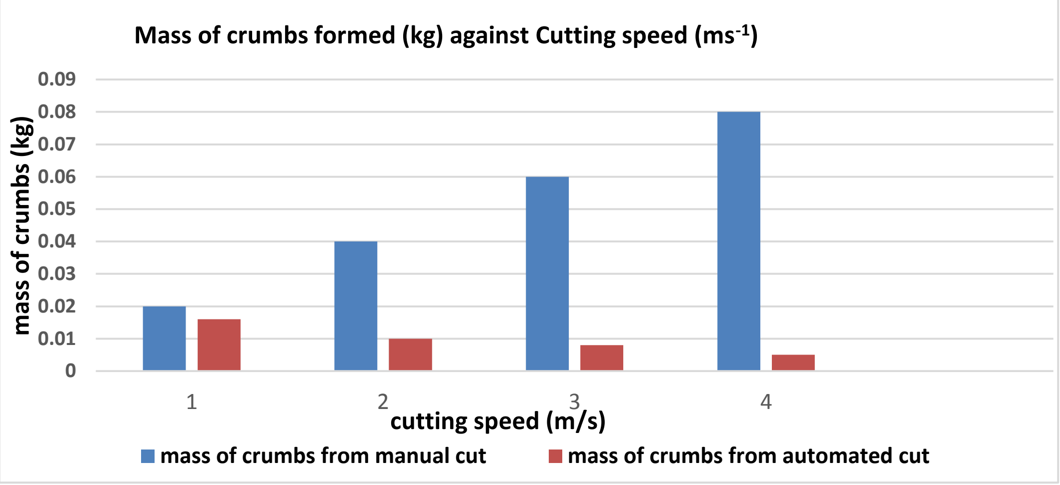

Fig. 1

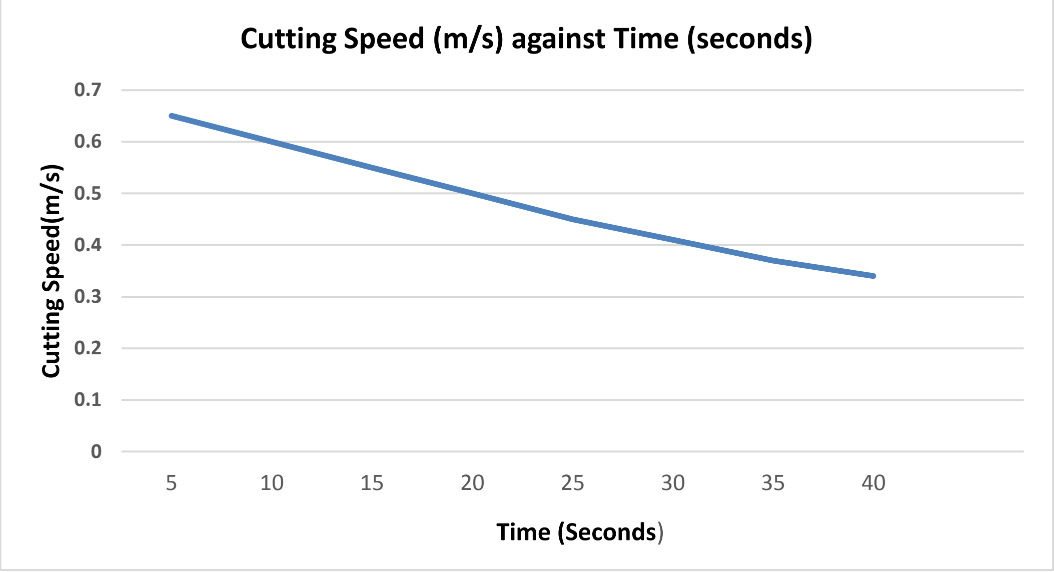

Fig. 2

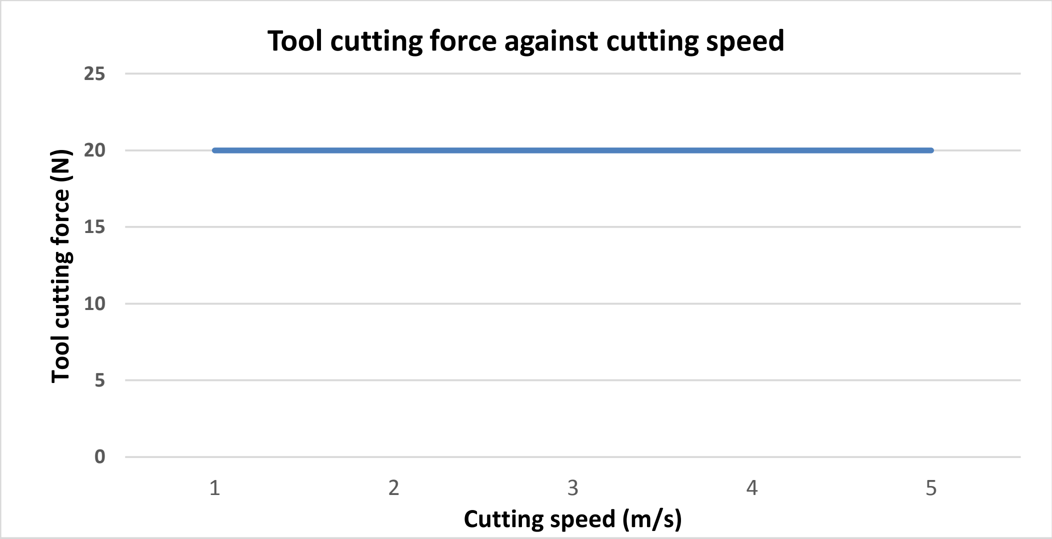

Fig. 3

4. DISCUSSION

A comparison of the manual and automated bread slicing machines showed the latter is faster in job processing, handles lesser material, provides better slicing, requires no skilled labour and works at a higher efficiency. This assertion comes from the fact that an automated bread slicer is modified technologically such that cutting speed can be varied at any point and a conveyor is incorporated to transit finished job. This means that the overall work rate is higher than that of a manual bread slicer. The bread guide and bread support are designed such that bread loaves are firmly held in position during the cutting operation. This implies a high cutting accuracy is achievable. The performance test revealed that bread loaves sliced manually are sometimes not as desirable as those sliced with automated machine owing mainly to lower cutting accuracy.

6. CONCLUSION

The performance of a manually operated bread slicing machine is of lesser efficiency when compared to the performance of an automated bread slicing machine. The higher overall system efficiency seen in an automated machine is mainly due to the modified feed and cutting mechanisms and introduction of a conveying mechanism. The cutting blades were installed in two sets of blade holders to optimize bread slicing. A vital feature is the speed selection system which ensured the cutting operation was performed more smoothly and this yielded comparatively much smaller bread crumbs at higher processing speeds. Materials selected are such that are available, machinable, corrosion resistant and able to withstand forces generated. They helped to achieve an efficiency of 96%. The loading capacity is only an insight than can be very useful in the construction of bigger models for commercial purposes.

REFERENCES

[1] ASTM F2 646-07 Standard Specifications for Bread slicing machines www.americanhistory.si.edu/collections/search/object/nmah_1317263

[2] Chapman, W.A.J. (1975). *Workshop Technology.* Edward Arnold Publishers Ltd. London.

[3] Hall et al (1971). *Machine Design.* McGraw-hill. London.

[4] Jain, R.K (2009). *Production Technology*. 17th Edition, New Delhi.

[5] Khurmi R.S and Gupta J.K. (2006). *Machine Design.* Chand S. and Company Ltd. New Delhi.

[6] Mubeen and Mubeen. (2004). *Material Science.* Khana Publishers, Delhi.

[7] Sharma P.C (1982). *Production Engineering.* Chand S. and Company Ltd., New Delhi.

[8] Shigley J.E. (1989). *Machine Design Fundamentals.* McGraw-hill Publishing Company, New York.